

Εφαρμογή (Διάσπαση Πεπερασμένων Αβελιανών Ομάδων)

Να βρεθούν όλες οι μη ισομορφείς αβελιανές τάξεις 600

ΛΥΣΗ

$$|G| = 600 = 2^3 \cdot 3 \cdot 5^2$$

$$k=3, p_1=2, p_2=3, p_3=5$$

$$n_1=3, n_2=1, n_3=2$$

$$\Pi(3)\Pi(1)\Pi(2) = 3 \cdot 1 \cdot 2 = 6 \text{ μη ισομορφείς}$$

$$1) \mathbb{Z}_{p_1^3} \times \mathbb{Z}_{p_2} \times \mathbb{Z}_{p_3^2} = \mathbb{Z}_8 \times \mathbb{Z}_3 \times \mathbb{Z}_{25} \cong \mathbb{Z}_{24} \times \mathbb{Z}_{25} \cong \mathbb{Z}_8 \times \mathbb{Z}_{75} \cong \mathbb{Z}_3 \times \mathbb{Z}_{200} \cong \mathbb{Z}_{600}$$

$$2) \mathbb{Z}_{p_1^3} \times \mathbb{Z}_{p_2} \times \mathbb{Z}_{p_3} \times \mathbb{Z}_{p_3} = \mathbb{Z}_8 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_5 \cong \mathbb{Z}_{24} \times \mathbb{Z}_5 \times \mathbb{Z}_5 \cong \mathbb{Z}_{120} \times \mathbb{Z}_5 \cong \mathbb{Z}_{40} \times \mathbb{Z}_3 \times \mathbb{Z}_5 \cong \mathbb{Z}_{40} \times \mathbb{Z}_{15} \cong \mathbb{Z}_8 \times \mathbb{Z}_{15} \times \mathbb{Z}_5$$

$$3) \mathbb{Z}_{p_1^2} \times \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2} \times \mathbb{Z}_{p_3^2} = \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_{25} \cong \mathbb{Z}_{12} \times \mathbb{Z}_2 \times \mathbb{Z}_{25} \cong \mathbb{Z}_{12} \times \mathbb{Z}_{50} \cong \mathbb{Z}_{300} \times \mathbb{Z}_2 \cong \mathbb{Z}_{100} \times \mathbb{Z}_2 \times \mathbb{Z}_3 \cong \mathbb{Z}_{100} \times \mathbb{Z}_6 \cong \mathbb{Z}_4 \times \mathbb{Z}_6 \times \mathbb{Z}_{25} \cong \mathbb{Z}_4 \times \mathbb{Z}_{150} \cong \mathbb{Z}_4 \times \mathbb{Z}_3 \times \mathbb{Z}_{50} \cong \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_{75} \cong \mathbb{Z}_2 \times \mathbb{Z}_{300}$$

$$4) \mathbb{Z}_{p_1^2} \times \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2} \times \mathbb{Z}_{p_3} \times \mathbb{Z}_{p_3} = \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_5 \cong \mathbb{Z}_{12} \times \mathbb{Z}_2 \times \mathbb{Z}_5 \times \mathbb{Z}_5 \cong \mathbb{Z}_{12} \times \mathbb{Z}_{10} \times \mathbb{Z}_5 \cong \mathbb{Z}_{60} \times \mathbb{Z}_{10} \cong \mathbb{Z}_{60} \times \mathbb{Z}_2 \times \mathbb{Z}_5 \cong \mathbb{Z}_{10} \times \mathbb{Z}_4 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \cong \mathbb{Z}_{30} \times \mathbb{Z}_4 \times \mathbb{Z}_5 \cong \mathbb{Z}_{30} \times \mathbb{Z}_{20} \cong \mathbb{Z}_{50} \times \mathbb{Z}_4 \times \mathbb{Z}_3 \cong \mathbb{Z}_{50} \times \mathbb{Z}_{12} \cong \mathbb{Z}_{20} \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \cong \mathbb{Z}_{20} \times \mathbb{Z}_6 \times \mathbb{Z}_5 \cong \mathbb{Z}_{15} \times \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_5$$

$$5) \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2} \times \mathbb{Z}_{p_3^2} = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_{25} \cong \mathbb{Z}_6 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_{25} \cong \mathbb{Z}_{150} \times \mathbb{Z}_2 \times \mathbb{Z}_2 \cong \mathbb{Z}_6 \times \mathbb{Z}_2 \times \mathbb{Z}_{50} \cong \mathbb{Z}_{50} \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_{75} \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_{150}$$

$$6) \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2} \times \mathbb{Z}_{p_3} \times \mathbb{Z}_{p_3} = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_5 \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_6 \times \mathbb{Z}_5 \times \mathbb{Z}_5 \cong \mathbb{Z}_{10} \times \mathbb{Z}_2 \times \mathbb{Z}_6 \times \mathbb{Z}_5 \cong \mathbb{Z}_{10} \times \mathbb{Z}_{10} \times \mathbb{Z}_6 \cong \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_{30} \times \mathbb{Z}_5 \cong \mathbb{Z}_{10} \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \cong \mathbb{Z}_{30} \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_5 \cong \mathbb{Z}_{30} \times \mathbb{Z}_2 \times \mathbb{Z}_{10} \cong \mathbb{Z}_{50} \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \cong \mathbb{Z}_{150} \times \mathbb{Z}_2 \times \mathbb{Z}_2 \cong \mathbb{Z}_{50} \times \mathbb{Z}_2 \times \mathbb{Z}_6$$

$$\cong \mathbb{Z}_{15} \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_5 \cong \mathbb{Z}_{30} \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_5 \cong \mathbb{Z}_{30} \times \mathbb{Z}_2 \times \mathbb{Z}_{10} \cong$$

$$\cong \mathbb{Z}_{75} \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2. \quad (\text{Με τον τρόπο αυτό υπάρχει πιθανότητα να ξεχάσουμε κάποιον σπώνασμο}).$$

Λόγω ότι στη αυτή η διαδικασία για ομάδες μεγάλων τάξεων (και συγκεκριμένα για $k \geq 3$) είναι επιπλέον απλώς όταν μας ζητούνται όλες οι μη ισομορφείς αβελιανές (στο συγκεκριμένο παράδειγμα θα γραφούμε):

$$(3, 1, 2), \mathbb{Z}_8 \times \mathbb{Z}_3 \times \mathbb{Z}_{25} \cong \mathbb{Z}_{600}$$

$$(3, 1, 1+1), \mathbb{Z}_8 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_5 \cong \mathbb{Z}_5 \times \mathbb{Z}_{120}$$

$$(2+1, 1, 2), \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_{25} \cong \mathbb{Z}_2 \times \mathbb{Z}_{300}$$

$$(2+1, 1, 1+1), \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_5 \cong \mathbb{Z}_{10} \times \mathbb{Z}_{60}$$

$$(1+1+1, 1, 2), \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_{25} \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_{150}$$

$$(1+1+1, 1, 1+1), \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_5 \cong \mathbb{Z}_2 \times \mathbb{Z}_{10} \times \mathbb{Z}_{30}$$

Σε περίπτωση που ζητάει ποια υψότερη είναι ισομορφική με κυκλικές ομάδες \mathbb{Z}_{600} , θα βρούμε ποια από τις προηγούμενες είναι υψότερη. Εδώ, είναι η $\mathbb{Z}_8 \times \mathbb{Z}_3 \times \mathbb{Z}_{25}$ όπου:

$$\mathbb{Z}_8 \times \mathbb{Z}_3 \times \mathbb{Z}_{25} \cong \mathbb{Z}_{24} \times \mathbb{Z}_{25} \cong \mathbb{Z}_8 \times \mathbb{Z}_{75} \cong \mathbb{Z}_3 \times \mathbb{Z}_{200}$$

Άλλες ισομορφικές ομάδες του \mathbb{Z}_{600} .